



# Mathematics Education and Graph Theory

Proceedings of International Seminar  
on Mathematics Education and Graph Theory  
June 9, 2014

**Editors:**  
Mustangin  
Abdul Halim Fathani

# **Mathematics Education and Graph Theory**

PROCEEDINGS OF INTERNATIONAL SEMINAR  
ON MATHEMATICS EDUCATION AND GRAPH THEORY



**Unit of Publication  
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Faculty of Teacher Training and Education  
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## **MATHEMATICS EDUCATION AND GRAPH THEORY**

**Proceedings of International Seminar on Mathematics Education and Graph Theory**

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Islamic University of Malang, 2014

*These proceedings contain the full texts of paper and talks presented  
in the International Seminar on Mathematics Education and Graph Theory  
on June 9, 2014*

### **Reviewers**

Surahmat Supangken (UNISMA, Indonesia)

Abdur Rahman As'ari (UM, Indonesia)

Kiki Ariyanti Sugeng (UI, Indonesia)

Sunismi (UNISMA, Indonesia)

Akhsanul In'am (UMM, Indonesia)

### **Editors**

Mustangin

Abdul Halim Fathani

### **Layouter**

Teguh Permadi

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Fax +62341-552249

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## PREFACE

These proceedings contain the full text of papers and talks presented in the International Seminar on Mathematics Education and Graph Theory. This seminar was held in conjunction with the International Workshop on Graph Masters. The workshop was held on June 7–8, 2014, while the seminar was on June 9, 2014. These events were organized by Islamic University of Malang (Unisma) in cooperation with Indonesian Combinatorial Society (InaCombS).

The workshop and the seminar would not have been possible without the time and energy put forth by the invited speakers. The invited speakers of the workshop were: **Mirka Miller**, University of Newcastle, Australia; **Joseph Miret**, Universitat de Lleida, Spain; **Christian Mauduit**, Institut de Mathematiques de Luminy, France; **Edy T. Baskoro**, Bandung Institute of Technology, Indonesia; **Surahmat Supangken**, Islamic University of Malang, Indonesia; **Tri Atmojo**, State University of Semarang, Indonesia; and **Purwanto**, State University of Malang, Indonesia.

The invited speakers of the seminar were: **Juddy Anne Osborn**, University of Newcastle, Australia and **Abdur Rahman As'ari**, State University of Malang, Indonesia. The seminar was held on the area of mathematics education and graph theory. The main themes of the mathematics education seminar include topics within the following areas (but not limited to): philosophy of mathematics education, curriculum development, learning methods and strategies, learning media, development of teaching material, and assessment and evaluation of learning. The main themes covered in graph theory seminar include topics within the following areas (but not limited to): degree (diameter) problems, ramsey numbers, cycles in graphs, graph labeling, dimensions of graphs, graph coloring, algorithmic graph theory, and applications of graph theory in various fields.

We would like to thank you to the invited speakers and all presenters who have submitted papers, for their valuable and inspiring presentation. A special appreciation goes to: **Surahmat Supangken**, Rector of Unisma and **Kiki Ariyanti Sugeng**, the President of InaCombS, who have made a lot of efforts to prepare this seminar.

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Malang, July 22, 2014

**Editors**



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# **Mathematics Education**

**PROCEEDINGS OF INTERNATIONAL SEMINAR  
ON MATHEMATICS EDUCATION AND GRAPH THEORY**

***Theoretical (Conceptual) Articles***

## RESEARCH INSTRUMENT DEVELOPMENT OF STUDENTS' REASONING PROCESS IN PROVING THEOREM

**Susanah**

Mathematics Department of Surabaya State University  
susanah.alfian@gmail.com

### Abstract

The goal of this paper is to exemplify a development of some research instruments in a qualitative study. The Research instruments were developed for collecting data in order to profiling reasoning processes of student in proving a theorem. The "4D model" of developmental research design was used in developing such instruments. Employing the first three steps of the 4D model, it was successfully developed three kinds of research instruments. The instruments consist of a mathematics ability test, a proving theormtask and interview guidance. The mathematics test is used to category student's ability, and the proving theormtask is used to know the student's reasoning. Meanwhile, the interview guide consists of some basic questions for assesing reasoning processes of student in proof of theorem. All of the instruments were valid since they had been validated by five experts andby two students through developmental testing.

**Keywords:** *Research Instrument, Reasoning Process, Proving A Theorem*

### INTRODUCTION

Reasoning ability is one of competencies required for students. This is because at all levels in the University that students are be able to find, create, and / or contribute to the development and practice of science and technology through research and scientific reasoning. (UURI No. 12 of 2012 on Higher Education in section 1 (b) Article 3, Article 13, Paragraph 3, Article 20 (1), Article 21 (1), Article 23 (1)).

Reasoning is an activity, a process or a thinking activity to draw a conclusion or make a new statement that is true based on some statements that the truth has been proved or assumed previously (Sadiq, 2004: 2). Lithner (2004: 406) stated that "reasoning is used to mean the line of thought, the way of thinking, adopted to produce assertions and reach conclusions." (Reasoning is a series of thought or way of thinking that is used to produce statements and to reach conclusion.)

Concerning on the importance of reasoning used in solving problems, students should be able to develop and improve their reasoning. By developing their reasoning, students are expected to learn how to analyze the situation, choosing

a strategy in solving the problem, drawing logical conclusions, developing and describing solutions, and recognizing how the solution can be applied. They should realize that by mathematical reasoning one can reflect on the settlement of a problem and to determine whether their settlements are reasonable or not.

One of activities that can show a person's ability of reasoning is to prove theorem. One of subjects that the material loaded with a discussion of theorem is geometry subject. Geometry is one of compulsory subjects that must be taken by undergraduate students of Unesa majoring in mathematics (Unesa Guidelines Books, 2012).

In studying geometry, students are expected to achieve the following skills (NCTM, 2000): (1) recognizing reasoning and proof as fundamental aspects, (2) making and investigating mathematical conjectures, (3) developing and evaluating mathematical arguments and proofs, and (4) selecting and using various types of reasoning and proving methods. According to French (2004) there are three major reasons why geometry is taught in schools and in universities, they are (1) expanding spatial awareness, (2) developing reasoning

skills, and (3) stimulating, challenging, and informing learners. In line with Duval (1998) said that the study of geometry includes three types of cognitive processes, each type has a function in the case of visualization, construction and reasoning. From some of the above quotes indicate that by studying geometry, reasoning skills and proving skills of students can be trained.

Research of Harold & Sowder (in ReyhanI, 2012) said that the ability to learn mathematics and mathematical reasoning abilities of students is not an easy task because, in general, the process is influenced by education, cultural, social and historical approach. It can be concluded that there is a relationship between mathematical reasoning and mathematical abilities, so that differences in math ability is likely to give different reasoning results. Students' reasoning that has higher mathematical abilities may be different from students' reasoning that has medium or low mathematical abilities in proving theorem. This is supported by the results of Siswono's study (2004) showed that students in the three groups (mathematics ability: high, medium and low) have a different strategy in constructing theorem and proving theorem.

In order to explore and investigate the process of students' reasoning in proving theorem, research tools that can be used to obtain such data need to be developed. Therefore it is necessary to develop research instruments of the profile of students reasoning process in proving theorems, while the research instrument is a math skills test, theorem proving task and interview guides.

Based on the background above, the writer raises a question in this study, that is: "How is the Process of Developing Research Instrument of Students' Reasoning Profile Students in Proving Valid Theorem?"

## LITERATURE REVIEW

### Reasoning

Logical thinking is thinking based on logic. According to Big Indonesian

Dictionary (2007) being logical means in accordance with the logic; true by reasoning; reasonable and logic is a knowledge of thinking's structure or thinking's way that makes sense. Therefore, the conclusions that follow the laws of logic do not necessarily correspond to reality. What the most important is that logical thinking is a way of thinking not truth of the base of thinking (starting thinking).

Peter and Yani (2002) argued that reasoning is derived from the word that means activity to think logically or ability / range of thinking. Reasoning is closely related to how humans reach certain conclusions from the premises either directly or indirectly. According to Keraf (1982: 5), reasoning or often also called way of thinking is "thinking process that seeks connection between the facts or evidences that are known leading to a conclusion." Suharnan (2005: 160) said that "reasoning is an activity to assess relationship between propositions that are arranged in the form of premises, and then determining the conclusion. Lithner (2004: 406) said the same thing that "reasoning is used to mean the line of thought, the way of thinking, adopted to produce assertions and reach conclusions." According to him, reasoning is a series of thought or way of thinking that is used to produce statements and to reach conclusions.

Based on some of the above opinions, reasoning contemplated in this study is thought to draw logical conclusions based on the statement that the truth has been proved or on the premises or information provided.

According to Soedjadi (2007) reasoning or also called higher-order thinking can be divided into three levels; namely, critical thinking, reflective thinking and creative thinking. According to Krulik and Rudnick (1995) reasoning is part of the thinking. Reasoning categorized into three levels; ie, basic thinking, critical thinking and creative thinking.

Based on the description of each category of reasoning mentioned before, it is showed that the category of creative reasoning in Soedjadi, Krulik and Rudnick, however, there are differences and

similarities in their descriptions. Category of critical thinking is only available in Soedjadi, Krulik and Rudnick. Category reflective thinking is only available in Soedjadi, in Krulik and Rudnick, is not categorized alone but it includes the description of critical thinking.

Lithner (2004) and Sumpter (2009) stated that to complete the task in data processing, there are four steps of reasoning structures used are: 1) Problematic Situation, 2) Strategy Awards. 3) Strategy implementation. And 4) Conclusion.

Which will be considered in this study is the sequence or steps used by students in proving theorems. The sequence or steps are:

- a. Understanding theorem  
Indicators used to determine that the students can understand theorem is showed by rewriting the theorem.  
What is meant by rewriting the theorem is writing about:
  1. What is known in theorem
  2. What is being asked in theorem or what needs to be proved?
- b. Determining Strategy  
The indicators used were selected proof in proving theorem.
- c. Setting up necessary components in a strategy that has been selected in proving theorem, the components can be in the form of:
  1. Fact, definition, axiom or theorem that has been proved previously.
  2. Sketch/picture aid (to construct concept)
- d. Implementing the chosen strategy
- e. Concluding proof result as being asked.

### Proving Theorem

In Big Indonesian Dictionary a proof means something that tells a truth of an event, so to prove is to show or to convince the truth of an event. According to Bell (1981: 290) "proof is any argument or presentation of evidence that convinces or persuades someone to accept a belief". Bell means that proof is some arguments or presentation of facts that convince someone

to believe. The quote above indicates that a proof can make someone believe an event whether it is true or not.

While the meaning of evidence in mathematics, one of which is given by Stylianides (2007) as follows: evidence is the basis for making and learning math. Evidence is the basis of mathematical understanding and critical to develop, build, and communicate mathematical knowledge and evidence is also an important part of increasing reasoning.

One of the most important tools in learning mathematics is evidence. Some views from De Villiers and Knuth, Tray, Lee, and Weber (in Kogce et al, 2010) demonstrated that the evidence played a role in mathematics as a means of verification, explanation, communication, invention, and systematization. Evidence of a role: (a) to verify that a statement is true, (b) to explain why a statement is true, (c) to communicate the knowledge of mathematics, (d) to find or create new mathematics, (e) to systematize statement be the axiom system, (f) truth guidance or falsity of an argument and relationship between justification, (g) justification of definition, (h) development of intuition, and (i) autonomy provision.

The mathematician in finding a theory or theorem has to take several processes that can convince previous mathematicians with the evidence presented. The first mathematician analyzes a problem or statement. Then, mathematician asks about whether the statement is true or false by looking at previous evidence and examines how that statement can be obtained by utilizing the theorem proof.

In mathematics, including geometry, the word proving often found in solving problems, questions or theorems. According to Travers, et al (1987: 3), "theorem is a statement that the truth must be proved while the proof is based on definitions, postulates, or theorems that have been demonstrated before." One of the requisite courses of theorem is geometry. Travers (1987: 2) said that "geometry studies the relationship point, line, plane, flat, and space. The geometry is a

mathematical system that uses logical reasoning, constructed from the facts that are known and acceptable to find new properties ". Following that Travers (1987: 2) stated that "geometry is the science that does not prioritize relationships between numbers although geometry using numbers." From the reviews that have been presented to prove the theorem in geometry is one form in solving the problem.

In solving problem, Polya (1973) used the following steps:

- 1) Understanding the Problem.
- 2) Devising a Plan.
- 3) Carrying Out the Plan.
- 4) Looking Back

In proving theorem according to Susanah (2002) there are some sequence that need to be followed. They are as the following:

- 1) Rewriting the theorem
- 2) Choosing the suitable theorem to be proved
- 3) Preparing concepts needed in proof
- 4) Implementing proving process based on what is known and concepts related to the theorem.
- 5) Concluding the result of proving

Based on the above opinion that will be observed is the reasoning used by students in proving theorem associated with triangle. As for the reasoning to be gained from students linked with the sequences or steps that students use in proving theorem that is reasoning:

- a. Understanding Theorem  
Criterion used to know the reasoning used by students in understanding theorem for example in (if):
  1. Rewriting the theorem.  
What is meant by rewriting here is writing about:
  2. What is known in proving?
  3. What is asked in proving or what is to be proved.
- b. Strategy choosing  
Criterion used is type of proving chosen in proving theorem. (For example direct or indirect proof)

- c. Preparing component needed in strategy chosen in proving theorem, the components can be in the form of:
  - 1) Fact, definition, axiom, or theorem that has been proving previously.
  - 2) Sketch/picture aid made (to construct concept)
- d. Implementing the chosen strategy
- e. Concluding the result based on what being asked.

### Reasoning in Proving Theorem

It has been described previously that the purpose of this study is the reasoning in logical thinking or thinking process to draw conclusions based on the assertion that the truth has been proven or based on the premises or the information provided. In proving theorem, reasoning used is deductive reasoning, so the argument used is deductive arguments. This is supported by the opinion of Heinze, A and Reiss, K (2006) that mathematical proof is deductive reasoning pattern. This means that for every conclusion in proof there are arguments in favor of the deductive character.

According to Bell (1981: 293) "deductive argument is a valid argument form that operates on a set of hypotheses that are assumed to have the correct value until it ends with a set of conclusions derived logically from the hypotheses." There are two general categories used in proving theorem with deductive argument that is direct and indirect arguments. Proof with direct argument is constructed from the hypothesis, whereas indirect proof of the argument is based on a statement that is accepted as true, because contraposition of the statement is true or because out of the fact it would result in a conflict with something that is considered correct.

Proving theorems by Travers (1987) can be done by direct evidence or by indirect evidence, because not all easy theorems proved by direct evidence although there is a theorem that can be proved by direct evidence or indirect evidence.

In this study what will be discussed is about deductive reasoning

used by students in proving theorems and explore why and how students use reasoning to prove theorem.

### **Correlation between Reasoning and Mathematics Ability**

Ability is defined as "the capability, skill, or strength of human (Big Indonesian Dictionary, 2007: 707). Ability is divided into two types, namely physical abilities and intellectual abilities. Physical ability is the ability to perform a task that requires effort, skill, strength, or similar characteristics. Intellectual capability is skills needed to perform mental activities (thinking, reasoning) to solve the problem. When proving a theorem, a student involves mental activity (thinking, reasoning) so intellectual ability is needed.

According to Schoenfeld (1985) a key capability is the source of knowledge and behavior (knowledge / skills base), heuristics (strategies of guesswork for non-standard problems), control (metacognition), and belief systems. Mathematical ability is the knowledge and basic skills needed to be able to perform mathematical manipulations including conceptual understanding and procedural knowledge. According Syaban (2009) matters that include understanding of a concept is the ability of reasoning, identifying and applying the principles, the ability to manipulate ideas about understanding the concepts in a variety of ways, and others. Whereas procedural knowledge is included in the complete system of equations, draw a circle in the triangle and so on. In other words, if someone has a good ability of reasoning hopefully understanding of the concept is also good.

It is also supported by Sausalito (2007) said that the students who used to use deductive reasoning on a regular basis will help them in learning the basic deductive principles (mathematics), so it will be easier for them to learn

From the opinions above reasoning indicates that a person's understanding is associated with the understanding of a concept being studied, so that there is a relationship between mathematical

reasoning and mathematics skills. It is alleged that differences in mathematics ability is likely to give results on different reasoning. Reasoning of students with high mathematical reasoning may be different from medium or low mathematics ability of student in proving theorem.

### **Research Instrument**

Measuring tool in research is usually called instruments. According Sugiyono (2012) "research instrument is an instrument used to measure the natural and social phenomena that are observed." Quantitative research instrument is in contrast to qualitative research. In quantitative research instruments are generally standard-made and has been designed clearly before heading to the field for collecting the data, but qualitative research instruments may change according to conditions on the field. Therefore, the main instrument of qualitative research is the researcher himself, but after the research focus is clear, the research instruments will be developed commonly called aided instruments that are expected to complement and compare data that has been discovered through observation and interviews.

Based on the description above, aided research instruments in this study will be developed which will be used to discover how the profile of students' reasoning process in proving theorem. In this study three aided instruments are developed, they are test of mathematics ability, mathematical proof tasks and interview guides.

#### **a. Mathematical Ability Test Instrument**

Mathematical ability test instrument was developed based on the analysis of early mathematics ability of students who have gained and associated with mathematics materials that have been acquired during the course in semester one and semester two. This instrument was given to the students after two semesters and students passed the geometry course. Based on Mathematics Education curriculum of Unsesa (guidebook, 2012) in the semester one students acquire Basic

Mathematics and Introduction to Basic Mathematics and in semester two obtain Geometry, Integral Calculus, Number Theory and Methods of Statistics. Mathematical Ability Test Instrument is adopted from the problems associated with the material SNMPTN exponents, systems of equations, polynomials, quadratic equations, statistics, ranks, functions, matrices, geometry, trigonometry, integral, combinations, and limit.

**b. Mathematical Proof Task of Theorem**

The mathematical proof task of theorem is used to discover students' reasoning in proving theorem. The reasoning here is logical thinking to make statement or to draw conclusion based on premises or information that has been given or statement that the truth has been proved. Students' reasoning will be explored based on reasons given at every steps used in proving theorem in their works. The steps are as the following; 1) Understanding theorem, 2) Determining strategy, 3) Preparing components needed, 4) Implementing the strategy, and 5) Concluding the proof.

**c. Interview Guide Instrument**

Interview guide is used to explore reasoning process done by research's subject based on their work in proving theorem. Questions asked are unstructured and are matched properly with statements, answers, and reasons given by students.

**Developmental Model According to Thiagarajan**

Developmental model according to Thiagarajandkk., (1974) consists of four steps called "Four-D Models" (Model 4-D). The steps are as the following:

- a. Defining consists of pre – post analysis, students's analysis, concept analysis, task analysis, learning objective specification.
- b. Designing consists of criterion based test, media chosen, format chosen, and pre design.
- c. Developing consist of expert validity and experiment
- d. Dessiminating consists of validation based on experiment

result, packaging, and field dissemination.

**The four steps above are explained as the following**

**d. Defining Step**

The objective of this step is to determine and define requirements of research instruments bu analyzing the purpose in limitation of material being developed.

Defining step consists of five main steps as follows: pre-post analysis, students analysis, material analysis, task analysis, research objective specification. The steps are explained below.

**1) Pre-Post Analysis**

Pre-Post analysis is an investigation on basic problem needed in developing research instrument to get proper research instrument.

**2) Students Analysis**

Students analysis is a discussion on students characteristic based on what has been accomplished in pre-post analysis. The characteristic is a prior knowledge and cognitive development of the students.

**3) Concept Analysis**

Concept analysis is done to identify, specify, and arrange concepts systematically that will be used to arrage instrument based on pre-post analysis.

**4) Task Analysis**

Task analysis is done to identify main abilities of students in doing the task based on concept analysis result.

**5) Specification of Objectives**

Specification of objective is in the form of explaining the task analysis result and concept into specific objective. Specific objective of learning will be used as the basis in making research instrument.

**b. Design**

This step aims to design research instrument prototype. It begins with the purpose of each instrument. The design step consists of four steps as follows.

**1) Criterion-testConstruction**

In this step, the researcher arranges

mathematics ability test, theorem proving task and interview guide. the instrument making is based on previous pre-post analysis, students analysis, and concept analysis.

2) Media Selection

Media selection is done by determining suitable media that will be used in developing instrument. Media selection process is matched with concept, task, and interview analysis.

3) Format Selection

Format selection in developing research instrument consists of format selection for content arrangement, strategy selection, and source used to make the instrument.

4) Initial Design

Initial design is defined as research instrument to be developed to get draft I.

**c. Develop**

The purpose of this step is to produce the revised research instrument draft based on expert comments and field experiment data. The activity is as the following.

1) Expert Appraisal

Expert appraisal consists of content validity that cover all research instruments has been developed in the design step. Validation result of the expert will be used as a basis to revise and complete the research instrument in the design step.

2) Developmental testing

Developmental testing is done to obtain information about response, reaction, comment from the students on the instrument.

**d. Disseminate**

In this step, the research instrument produced in developmental testing in wider scale to collect the data. However, in this research disseminating step is not done yet due to time limit.

**METHOD**

**Research Design**

Based on the research question proposed previously, this study used developmental research as the research design. What to be developed is the research instrument that cover Mathematics Ability Test, Theorem Proving Task, and Interview Guide instrument.

**Research Instrument Developmental Procedure**

Developmental procedure of the research instrument is done by following four D-Model (4D) teaching instrument but is modified according to researcher's need. The procedure is described as follows.

**Step: Defining**

a. Pre-Post Analysis

The basic problem is determined in this step needed in developing research instrument. Investigation in this step is done on relevant theory, for example mathematical ability, reasoning, proving, and theorem, so a suitable instrument is found to be developed. In this research, the developing instrument consists of Mathematical Ability Test, Theorem Proving Task, and Interview Guide.

b. Students Analysis

Students analysis is an investigation about students characteristics related to research instrument development which is determined in pre-post analysis. This characteristic contains prior mathematical ability of students before and during study, and students' cognitive development.

c. Material Analysis

In this step main material identification is done particularly material that the students has mastered before and during study in semester one and semester two.

d. Task Analysis

In this step task given to students is identified based on material analysis. Task planning given to students is proving theorem.

e. Specification of Objectives

specification of objective activity is an activity that explains task analysis and concept analysis into specific objective. This research does not mention learning objective, but the indicator and specific objective are made as a basis in making the instrument.

**Step: Design**

1. Media Selection

Media used in developing instrument of this research is visual and auditory media.

2. Format Selection

Format selection is a format to develop instrument in this research that consists of format selection to make Mathematical Ability Test, Theorem Proving Task, and Interview Guide instruments.

3. Initial Design

Activity done in this step is making draft of instrument Mathematical Ability Test, Theorem Proving Task, and Interview Guide that the so called Draft I.

**Step: Develop**

The final output of developing step is research instrument draft. Therefore, activity done after getting Draft I is as the following:

a. Expert Validation

Validation is done on research instrument of Draft I. This validation is carried out by experts, they are lecturers of geometry, lecturers of evaluation, and lecturers of other mathematics subjects. The validation focused on the content and language. The result of validation is used as a basis to revise and complete the research instrument.

e. Field Test

Field test is done to obtain information about reaction and comments from students on the instruments.

**Step: Disseminate**

In this step the research instrument as a result of this study will be used in a wider range to collect data. However, dissemination is not done yet due to time limit. The scheme of research instrument validation is as the following:

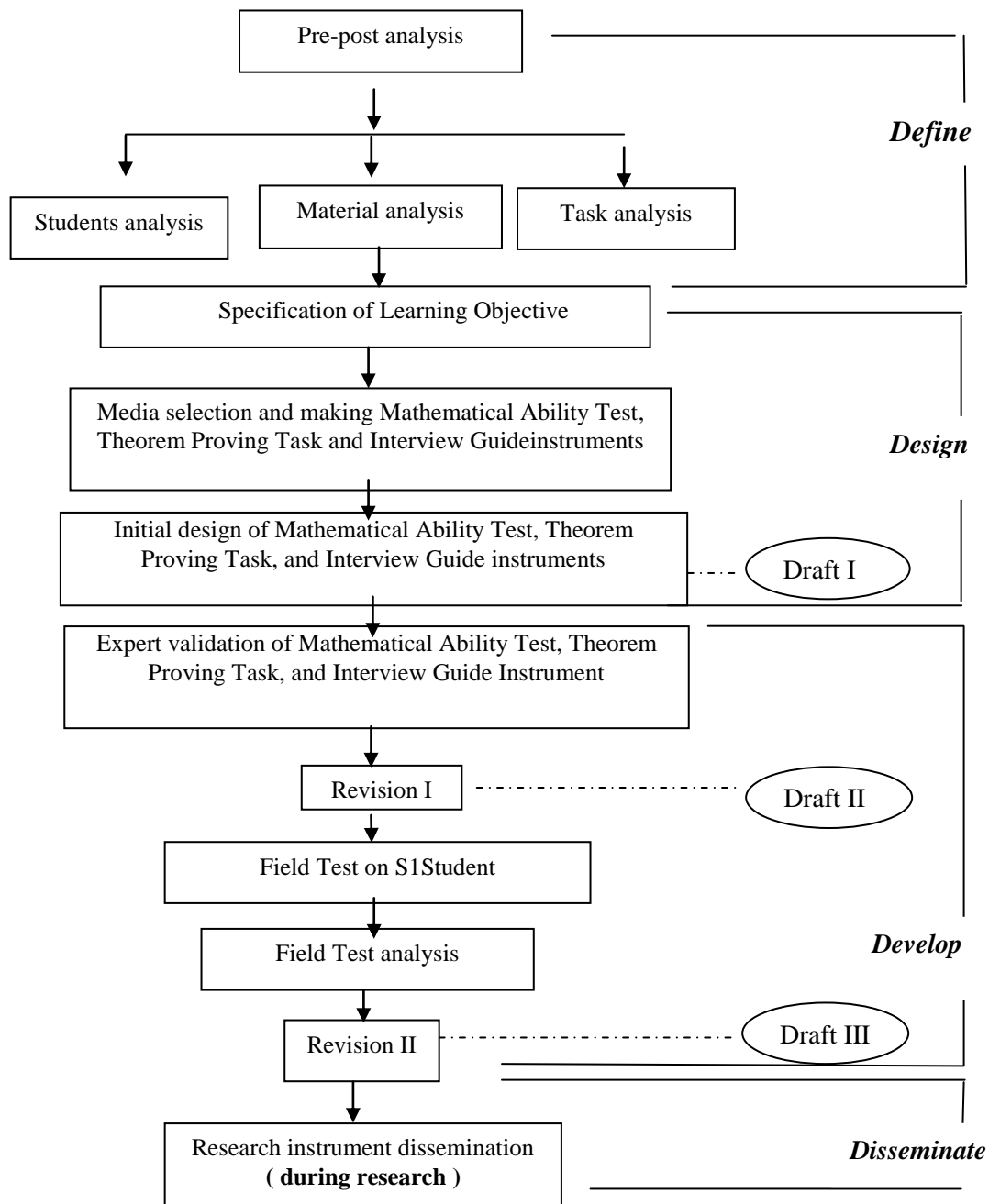


Figure 1. Steps in Developing Research Instrument Based on Thiagarajan 4-D model.

## DISCUSSION

### Research Result

As explains in research method that the development of research instrument is made and developed by following three first steps of Thiagarajan “4D model”, such

as define, design, and develop. The steps are explained as the following.

### Step: Define

#### a. Pre-Post Analysis

The basic problem is determined in this step needed in developing research instrument. Investigation in this step is

done on relevant theory, for example mathematical ability, reasoning, proving, and theorem, so a suitable instrument is found to be developed. In this research, the developing instrument consists of Mathematical Ability Test, Theorem Proving Task, and Interview Guide.

b. Students Analysis

Students analysis is an investigation about students characteristics related to research instrument development which is determined in pre-post analysis. This characteristic contains prior mathematical ability of students before and during study, and students' cognitive development.

c. Material Analysis

In this step identification of main material that has been mastered by students is done. The instrument is given to semester two students that has passed geometry course. Based on Mathematics Department's Curriculum of Unesa (Guide Book, 2012) in semester one students get Basic Mathematics and Introduction to Basic Mathematics courses and in semester two they get Geometry, Integral Calculus, Number Theory and Statistics Method. Based on the analysis done the material that will be used is material related to exponents, systems of equations, polynomials, quadratic equations, statistics, ranks, functions, matrices, geometry, trigonometry, integral, combinations, and limit. For theorem proving task, triangle-related theorem will be chosen.

d. Task Analysis

In this step task given to students is identified based on material analysis. Task planning given to students is proving theorem.

e. Specification of Objectives

Specification of learning objective is an activity that explains the result of task and concept analysis becoming specific objectives. This research does not use learning objective because this research does not develop teaching instrument. As such, the objective used in making research instrument based on

material analysis and it is explained into indicators or instrument making objective. Research instrument draft I consists of 13 question items of mathematical ability test, two equivalent theorem and one set interview guide.

**Step: Design**

a. Media Selection

The medium used to develop the instrument in this study is a visual and audiotape. This media is used to investigate student reasoning process based on the results of theorem proving task that has been done.

b. Format Selection

What is meant by selecting format is to develop research instrument in this study. It consists of format selection for Mathematical Ability Test, Theorem Proving Task, and Interview Guide. Format used in mathematical ability test is 13 essay questions, and theorem is selected as the task is triangle theorem and one set interview guide instrument which is unstructured.

c. Initial Design

Activity done in this step is arranging mathematical ability test instrument, theorem proving task, and interview guide.

**Step: Develop**

The final purpose of this step is resulted a draft of research instrument. Therefore, activity done after getting draft 1 is:

a. Expert Validation

Validation is done for research instrument draft 1. It is done by 2 lecturers of geometry, 1 lecturer of evaluation, and 2 lecturers of other mathematics lessons. Validation focused on the content and language. The result of validation is used as a basis to revise and complete the research instrument. The result of validation is presented below.

**Assessment aspect of mathematical ability test:**

- Assessment on the content of material
  - 1) Question item is matched with the indicator.
  - 2) Question item and answer is as expected.
  - 3) Time needed to answer question is proportional
- Assessment on the language of the question
  - 1) Language used in the question is matched with the principle and standard use of Indonesian Language.
  - 2) Question formulation use word/sentence with no bias or misunderstanding
  - 3) Instruction of the question is clear and communicative.

**Assessment aspect of theorem proving task:**

- Assessment on the content of material

- 1) Theorem item aims to discover understanding about proving theorem
- 2) Theorem item can investigate reasoning which is used by students
- 3) Theorem is chosen properly for students S1.

- Assessment on the language of task
  - 1) Language used in theorem is matched with the principle and standard use of Indonesian language.
  - 2) Theorem formulation use word/sentence with no bias or misunderstanding
  - 3) Instruction of the task is clear and communicative.

While **assessment aspect of interview guide** is only done on its language its self.

- Assessment on the language
  - 1) The question used is correctly matched with the principle and standard use of Indonesian Language.
  - 2) The question used has no bias or misunderstanding.
  - 3) The instruction is clear and communicative.

**Assessment Criteria of Mathematical Ability Test, Theorem Proving Task, and Interview Guide instruments:**

V	: valid	( if the three criteria are achieved)
CV	: valid enough	(if the two criteria are achieved)
KV	: less valid	(if one criteria is achieved)
TV	: Invalid	(if no one criteria is achieved)

TR :can be used without revision  
 RK :can be used with minor revision  
 RB :can be used with major revision  
 TD :cannot be used

Table 1: Assessment Recapitulation of Mathematical Ability Test

No. question	Frequency of validator statement on the content of material				Frequency of validator statement on the language if question				Conclusion			
	V	CV	KV	TV	V	CV	KV	TV	TR	RK	RB	TD
1.	1	3	1		4	1			4	1		
2.	5				5				5			
3.	4	1			4	1			4	1		
4.	5				5				5			

5.	4		1		4		1		4	1
6.		4	1			5			4	1
7.	5				5				5	
8.	5				5				5	
9.		4	1			4	1			5
10.	5				5				5	
11.	5				5				5	
12.	5				5				5	
13.	5				4	1			4	1

Table 2: Assessment Recapitulation of Theorem Proving Task Instrument

Type of Task	Frequency of validator statement on the content of material				Frequency of validator statement on the language of question				Conclusion			
	V	CV	KV	TV	V	CV	KV	TV	TR	RK	RB	TD
1.	5				5				5			
2.	5				5				5			

Table 3: Assessment Recapitulation of Interview Guide Instrument

Assessment on the language of question				Conclusion			
V	CV	KV	TV	TR	RK	RB	TD
5				5			

Based on Table 1 until Table 3 above, it can be concluded that research instrument is mathematical ability instrument, theorem proving task, and interview guide. Those instruments have achieved assessment criteria although in language and material aspect there are still minor revision.

b. Field Test

Field test is done on two students who have passed geometry course, but they are not the subject of the study. Based on the result of field test, the instrument readability can be used in the next dissemination step.

**CONCLUSION**

Based on the question and the discussion have been explained before, it can be concluded that the process of research instrument development of the students' reasoning profile in proving theorem has produced three research

instrument, they are Mathematical Ability Test, Theorem Proving Task, and Interview Guide. Instrument development refers to Thagarajan 4D model. The three first step are generally valid and can be used with minor revision by validator and base on field test the instrument can be used.

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